§3.4. Differentrability (§11.4)

#3. $z=\sqrt{4-x^2-2y^2}$, find an eq of the tangent plane at $(1,-1,1)$

sol: $z_x=\dfrac{-x}{\sqrt{4-x^2-2y^2}}\Big|_{(1,-1)}=-1$

$z_y=\dfrac{-2y}{\sqrt{4-x^2-2y^2}}\Big|_{(1,-1)}=2$

$\therefore \vec{n}=\langle -1,2,-1\rangle \Rightarrow$ eq: $x-2y+z=4$ ※

#5. $z=y\cos(x-y)$ 同上在 $(2,2,2)$ 点求切平面 eq

sol: $z_x=-y\sin(x-y)\Big|_{(2,2)}=0$

$z_y=\cos(x-y)+y\sin(x-y)\Big|_{(2,2)}=1$

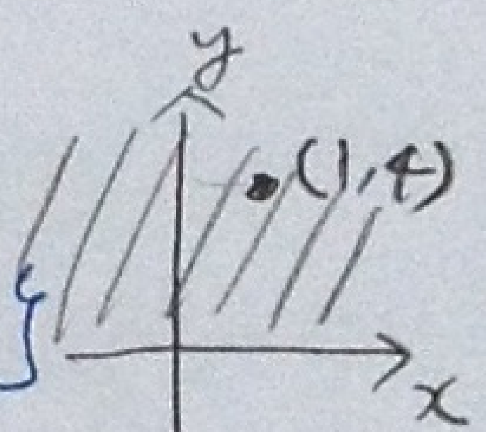
$\therefore \vec{n}=\langle 0,1,-1\rangle$, eq: $y-z=0$ or $y=z$ ※

11–14. Explain why 可微在指定的点 Find the $L(x,y)$ of the fct at that pt.

#11. $f(x,y)=x\sqrt{y}$, $(1,4)$

sol: $f_x=\sqrt{y}$ and $f_y=\dfrac{x}{2\sqrt{y}}$

Thus f_x and f_y are conti. on $\Omega=\{(x,y)\mid y>0\}$



$\because (1,4)\in\Omega$ by Thm, f is differentiable at $(1,4)$. □

$f_x(1,4)=2$, $f_y(1,4)=\frac{1}{4}$

$\therefore L(x,y)=f(1,4)+f_x(1,4)(x-1)+f_y(1,4)(y-4)$

$=2x+\frac{1}{4}y-1$ ※ (or $L(x,y)=2x+\frac{1}{4}y-1$)

$f(1,4)=2$,

#13 $f(x,y)=\tan^{-1}(x+2y)$, $(1,0)$

sol: $f_x=\dfrac{1}{1+(x+2y)^2}$, $f_y=\dfrac{2}{1+(x+2y)^2}$, $\forall (x,y)\in\mathbb{R}^2$

$(x+2y)^2+1>0$

Both are conti on $\mathbb{R}^2$, so f is differentiable at $(1,0)$ □

$f_x(1,0)=\frac{1}{2}$, $f_y(1,0)=1$ $f(1,0)=\tan^{-1}1=\frac{\pi}{4}$

$\therefore L(x,y)=\frac{1}{2}x+y+\frac{\pi}{4}-\frac{1}{2}$ ※

#15. Find the linear appro. of fct $f(x,y)=\sqrt{20-x^2-7y^2}$ at $(2,1)$ and use it to estimate $f(1.95, 1.08)$

sol: $f_x = \dfrac{-x}{\sqrt{20-x^2-7y^2}}\Big|_{(2,1)} = -\dfrac{2}{3}$, $f_y = \dfrac{-7y}{\sqrt{20-x^2-7y^2}}\Big|_{(2,1)} = -\dfrac{7}{3}$

$f(2,1)=3$

$\therefore L(x,y) = 3-\frac{2}{3}(x-2)-\frac{7}{3}(y-1)$

$f(x,y) \underset{1\text{階近似}}{\approx} L(x,y)$

$f(1.95, 1.08) \approx 3-\frac{2}{3}(-0.05)-\frac{7}{3}(0.08) = \frac{8.54}{3} = 2.84\bar{6}$ #

#19. Find the differential of $z = x^3\ln(y^2)$

sol: $z_x = 3x^2\ln(y^2)$, $z_y = x^3\dfrac{2y}{y^2} = \dfrac{2x^3}{y}$

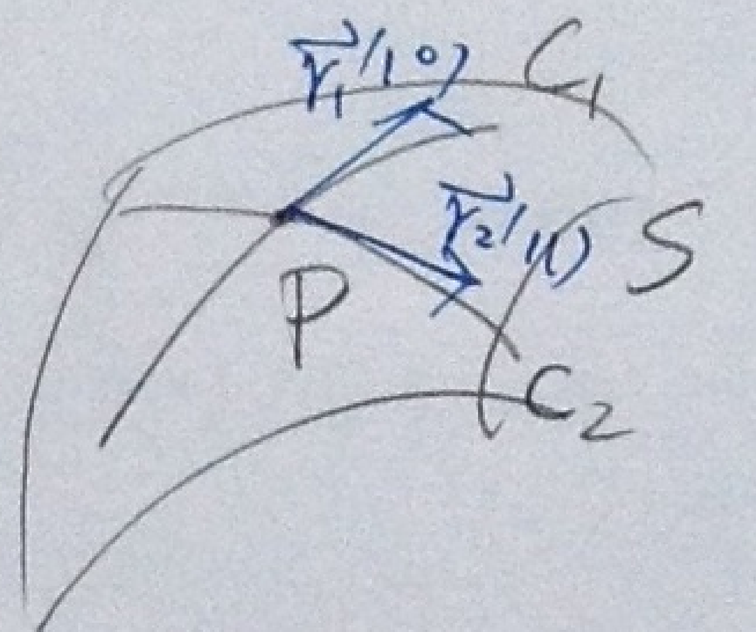
$\Rightarrow dz = 3x^2\ln(y^2)\,dx + \dfrac{2x^3}{y}\,dy$ #

#32. Surface S 上有兩條 curves

$\vec{r_1}(t) = \langle 2+3t,\ 1-t^2,\ 3-4t+t^2 \rangle$ 及

$\vec{r_2}(u) = \langle 1+u^2,\ 2u^3-1,\ 2u+1 \rangle$

求過 $P(2,1,3)$ 曲面 S 之切平面 eq



sol: $\begin{cases} 2+3t=2 \\ 1-t^2=1 \\ 3-4t+t^2=3 \end{cases} \Rightarrow t=0$, $\vec{r_1}(0)=P$

$\begin{cases} 1+u^2=2 \\ 2u^3-1=1 \\ 2u+1=3 \end{cases} \Rightarrow u=1$, $\vec{r_2}(1)=P$

$\vec{r_1}'(t) = \langle 3, -2t, -4+2t \rangle$, $\vec{r_1}'(0) = \langle 3, 0, -4 \rangle$

$\vec{r_2}'(u) = \langle 2u, 6u^2, 2 \rangle$, $\vec{r_2}'(1) = \langle 2, 6, 2 \rangle$, 取 $\vec{v_2} = \langle 1,3,1 \rangle$

$\vec{n} = \vec{r_1}'(0) \times \vec{v_2} = \langle 12, -7, 9 \rangle$

$\therefore$ eq: $12x-7y+9z=44$ #

§3.4. Differentiability

#36. (a) The fct $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$

Show that $f_x(0,0)$ and $f_y(0,0)$ both exist but f is not differentiable at $(0,0)$

(b) Explain why f_x and f_y are not conti. at $(0,0)$.

Pf(a) 上課有講，有2个方法說明 f 在 $(0,0)$ 不可微

$f_x(0,0) = \lim\limits_{h\to 0} \dfrac{f(h,0)-f(0,0)}{h} = \lim\limits_{h\to 0} 0 = 0$ and $f_y(0,0) = \lim\limits_{h\to 0} \dfrac{f(0,h)-f(0,0)}{h} = 0$

Thus $f_x(0,0)$ and $f_y(0,0)$ both exist.

$C_1 : (x,0)$, $x>0$ $\quad f|_{C_1} = f(x,0) = 0 \longrightarrow 0$ as $x \to 0$

$C_2 : (x,x)$, $x>0$ $\quad f|_{C_2} = f(x,x) = \dfrac{x^2}{2x^2} = \dfrac{1}{2} \to \dfrac{1}{2}$ as $x \to 0$

$0 \neq \frac{1}{2}$ $\therefore \lim\limits_{(x,y)\to(0,0)} f(x,y)$ DNE, and hence $f(x,y)$ is not conti. at $(0,0)$, so f is not diff. at $(0,0)$. <法二>見上課筆記

~~Pf(b) Thm 之否逆命題，~~

Pf(b) 若使用Thm之否逆命題，只可得出 f_x or f_y is not conti. at $(0,0)$. 因此 explain why 兩者在 $(0,0)$ 均不conti.，只能按def.，首先求出 f_x and f_y，如下

$f_x(x,y) = \begin{cases} \dfrac{y(y^2-x^2)}{(x^2+y^2)^2}, & \text{if } (x,y) \neq (0,0) \\ \underset{\text{(a)已做了}}{0} & \text{if } (x,y) = (0,0) \end{cases}$; $f_y(x,y) = \begin{cases} \dfrac{x(x^2-y^2)}{(x^2+y^2)^2}, & \text{not } (0,0) \\ 0 & , (0,0) \end{cases}$

If we approach $(0,0)$ along the y-axis and $y>0$, then

$f_x(x,y) = f_x(0,y) = \dfrac{y^3}{y^4} = \dfrac{1}{y} \longrightarrow \infty$ as $y \to 0^+$

Thus $\lim\limits_{(x,y)\to(0,0)} f_x(x,y)$ DNE and hence $f_x(x,y)$ is Not conti. at $(0,0)$.

Similarly, we also get $f_y(x,y)$ is Not conti. at $(0,0)$.